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Low Probability of Intercept Based Multicarrier Radar Jamming Power Allocation for Joint Radar and Wireless Communications Systems

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Abstract: Due to the high demand for new wireless services and shortage of available radio frequency (RF) spectrum, joint radar and communication system is considered as a coexistence solution to the RF spectrum congestion problem. Therefore, joint radar and communication system has become an attractive platform for target parameter estimation. In this paper, the problem of low probability of intercept (LPI) performance based orthogonal frequency division multiplexing (OFDM) radar jamming power allocation is addressed for a joint radar and communication system. Given the knowledge of the radar transmitted signal, the communication signal, the channel impulse responses and the propagation losses of the corresponding channels provided by the jammer, three different LPI based criteria for radar noise jamming power allocation are proposed, whose purposes are to minimize the total noise jamming power by optimizing the multicarrier jamming power allocation while the achieved mutual information (MI) between the received echoes and the target impulse response is enforced to be less than a predefined threshold. The presented optimization problems are solved analytically and their solutions represent the optimum power allocation for each subcarrier in the OFDM jamming waveform. Numerical simulations show that the LPI performance of the jammer is considerably improved by the proposed strategies.

1. Introduction

1.1. Background and Motivation

Due to its significant enhancement for military operations, low probability of intercept (LPI) is an important part that needs to be taken into account in designing radar systems [1]–[3]. Several technical and tactical measures can be taken to achieve better LPI performance, such as power management, emission time control, maximum bandwidth, and electronic order of battle exploitation [4].

For a general radar system, the signal-to-interference-plus-noise ratio (SINR) and mutual information (MI) between the reflected radar return and the target impulse response should be maximized to obtain the best target detection performance and parameter estimation accuracy, respectively. However, from the point of view of the opponent of a radar, the jammer would like to minimize the SINR and MI to protect the target from detection and estimation by utilizing noise jamming. The increasing development of digital radio frequency (RF) memory and digital signal

processing has pushed for smart radar noise jamming that can be adopted against modern radar systems. Recent years have witnessed a growing interest on the radar jamming design, which has been extensively studied from various perspectives, and some of the noteworthy works include [5]-[8]. In [5], the SINR-based and MI-based jamming waveform design methods are proposed, which reduce the SINR and MI of the radar system, respectively. Later, the minimax robust jamming is addressed based on the SINR and MI criteria [6], where the radar waveform spectrum lies in an uncertainty class confined by known upper and lower bounds. It is demonstrated that the two criteria lead to different optimal jamming results but they have a close relationship from the Shannon's capacity equation which provides useful guidance on jamming power allocation for different jamming tasks. Song et al. investigate the interaction between a smart target and a smart multiple-input multiple-output (MIMO) radar from a game theory perspective [7], which is modeled as a two-person zero-sum game. The unilateral, hierarchical, and symmetric games are studied based on the available information set for each player, and the equilibria solutions are derived. Reference [8] presents a novel power allocation game model between a radar network and multiple jammers, where the objective of the radar network is to minimize the total transmitting power by the radars while achieving a given detection performance for each of the targets, while the intelligent jammers have the ability to observe the radar transmitted power and consequently decide its jamming power to maximize the interference to the radar network. In electronic warfare, the LPI design is also an essential and topical part of the jammer system. This is because high jamming power would lead to the hostile anti-radiation missile (ARM) attack, while the studies which investigate the LPI based radar jamming design are very limited. Shi et al. address the LPI based radar jamming waveform design for the first time [9], whose objective is to minimize the total jamming power while the achievable system performance outage probability is enforced to be greater than a specified confidence level, and the fuzzy chance-constrained programming (FCCP) is utilized to describe the complexity and uncertainty of overall system performance.

With the increasing demand for spectrum resources, the joint radar and wireless communications systems have been proposed as a coexistence solution to the RF spectrum congestion problem. In such joint systems, the radar and communication systems operate in the same bandwidth, without causing too much interference to each other. Bica et al. in [10] propose the orthogonal frequency division multiplexing (OFDM) radar waveform optimization algorithms in spectrum sharing environment based on two different applications, target characterization and target detection, where the scattering off the target due to the communication signals is considered as interference in the objective functions. Furthermore, the work is extended in [11] that the communication signals scattered off the target can be exploited at the radar receiver, which significantly improves the target detection performance for the radar system. Therefore, the joint radar and communication system has become an attractive configuration for target parameter estimation. However, on the basis of the research mentioned above, the problem of LPI based adaptive OFDM radar noise jamming power allocation in the joint radar and wireless communications systems, which has not been considered, needs to be investigated.

1.2. Major Contributions

The major contributions of this paper are fourfold:

(a) *By incorporating the radar transmitted signals, the communication signals, the channel impulse responses and the propagation losses of corresponding channels into our system model, we analytically derive the expression of the MI between the received echoes from the target and the target impulse response to provide a metric for the target characterization performance in the*

joint radar and communication system. In this paper, the jamming power allocation strategies are developed based on the target parameter estimation accuracy.

(b) *We address the problem of LPI based adaptive multicarrier radar noise jamming power allocation in the joint radar and communication system.* It is assumed that the knowledge of the radar transmitted signals, the communication signals, the channel impulse responses and the propagation losses of corresponding channels are intercepted and perfectly estimated by the jammer [11]. Various LPI based jamming power allocation algorithms are proposed, which minimize the total transmitting power of the jammer by optimizing the noise jamming power allocation while the achieved MI is enforced to be less than a given threshold. These criteria are different from each other in the way the scattering off the target due to the communication signals is considered as useful energy, as interference, or ignored altogether at the radar receiver.

(c) *All the multicarrier radar noise jamming power allocation strategies are formulated and solved analytically, where the method of Lagrange multipliers is employed to solve these problems.* Implementing the resource-aware techniques into real-time systems necessitates the quick and efficient allocation of the power resource. In this paper, the approach of Lagrange multipliers is adopted to solve the radar waveform design problems. It is shown in [11] that the approach of Lagrange multipliers can be extremely effective in solving multidimensional problems with many simple constraints such as lower and/or upper bounds on the variables.

(d) *The numerical results are provided to demonstrate the effectiveness of the proposed jamming power allocation strategies via Monte Carlo simulations.* We also reveal the relationships between the jamming power allocation results and the following three factors: radar transmitted signals, the communication signals, and the target impulse responses.

1.3. Outline of the Paper

The remainder of this paper is organized as follows. The considered joint radar and communication system model is introduced in Section II. In Section III, the LPI based adaptive radar noise jamming power allocation criteria are proposed and the associated optimization problems are formulated and solved analytically. Numerical results are provided in Section IV to demonstrate the effectiveness of the presented jamming power allocation strategies and analyze the effects of several factors on the allocation results. Finally, the conclusion of the paper is made in Section V.

Notation: The continuous time domain signal is denoted by $x(t)$; $x[k]$ is the associated sampled discrete time domain signal; and the frequency domain representation of a discrete sample $x[k]$ is $X[k]$. A single lower case bold letter \mathbf{x} represents a column vector with given dimension. By x_k we denote the k th element of vector \mathbf{x} . The symbol \otimes signifies the convolution operator. The superscript $(\cdot)^T$ and $(\cdot)^*$ indicate transpose and optimality.

2. System Model

Consider a joint radar and communication system with one monostatic radar and multiple communication base stations (BSs) aiming at tracking a point target [11], as depicted in Fig.1. We assume that the target radar cross section (RCS) follows a Swerling II model, in which the fluctuations are independent from pulse to pulse. The jammer attempts to protect the target by jamming the radar system under the assumption that the radar transmission waveform has been intercepted and perfectly estimated. Here, we will concentrate on the stand-off jamming. However, the derivations and the results can be straightforwardly extended to the self-protection jamming mode. The differ-

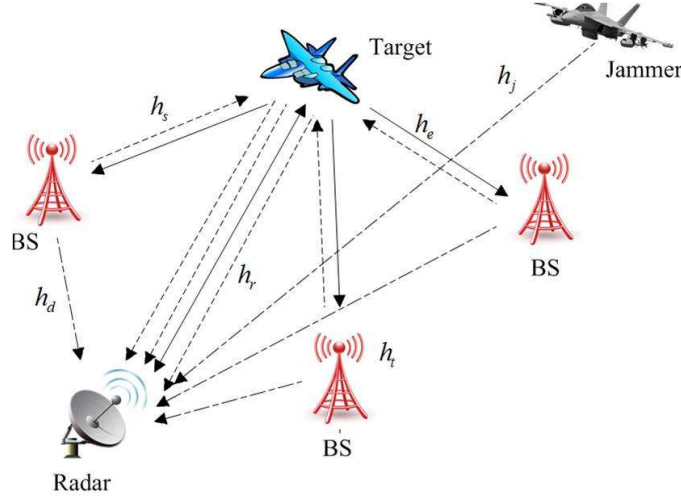


Fig. 1. Joint radar and communication systems model.

ence between these two jamming modes is the gain of the radar's transmit antenna in the direction of the jamming system, which is the side lobe antenna gain for the stand-off jamming and the main lobe antenna gain for the self-protection jamming, respectively. In the joint radar and wireless communications systems, the radar coexists with wireless communication systems in the same frequency band to increase the spectral efficiency. The radar receives the echoes scattered from the target due to the transmitted radar signals as well as the communication signals from the BSs, via two channels: a direct path and a path which is due to scattering off the target. The communication system carries out its task of data transmission by broadcasting signals throughout the space. In addition, we assume that the radar antenna is directional and oriented towards the target, thus the radar signal does not arrive at the communication systems through a direct path, but only scattered off the target, and therefore it does affect the communication systems.

In case of a monostatic radar, N_t communication systems and a jammer, the equation for the received signal at the radar can be expressed in continuous time as:

$$y(t) = r(t) + \sum_{i=1}^{N_t} [r_{s_i}(t) + s_i(t)] + r_j(t) + n(t), \quad (1)$$

where $y(t)$ denotes the received signal at the radar receiver, $r(t)$ is the echo from the target due to the radar transmitted signal, $r_{s_i}(t)$ is the scattering off the target due to the communication signal corresponding to the i th BS, $s_i(t)$ is the communication signal arriving through a direct line of sight path at the radar receiver corresponding to i th BS. $r_j(t)$ denotes the jamming signal received at the radar system and $n(t)$ stands for the additive white Gaussian noise. Without loss of generality, we will focus on a single communication BS. It is assumed that the radar, the wireless communication systems and the jammer use OFDM-type multicarrier signals with K subcarriers.

In this paper, the channels of interest are given as follows: h_r for the radar-target-radar path, h_e for the radar-target-BS path, h_s for the BS-target-radar path, h_d for the direct BS-radar path, h_t for the communication inside a BS cell, h_j for the jammer-radar path. The communication signal $x_s(t)$ is supposed to be deterministic and known at the radar receiver after a previous estimation step. We assume that the channels are stationary over the observation period. The channels $h_r(t)$, $h_s(t)$, and $h_e(t)$, corresponding to the target scattering, the jamming channel $h_j(t)$, as well as the

communication channels $h_d(t)$ and $h_t(t)$ are considered random and only known statistically. The radar channel impulse response is assumed to be a wide sense stationary Gaussian process. Thus, for a single communication system (1) can be rewritten as:

$$y(t) = x_r(t) \otimes h_r(t) + x_s(t) \otimes h_s(t) + x_s(t) \otimes h_d(t) + x_j(t) \otimes h_j(t) + n(t). \quad (2)$$

In this paper, it is assumed that both impulse responses $h_r(t)$ and $h_s(t)$ partly contain information about the target, which is because that the radar signals and the communication signals illuminate a common area of the target [11].

3. Problem Formulation

3.1. Basic of the Technique

The jamming power allocation strategy based on LPI can be formulated as: given the knowledge of the radar transmitted signals, the communication signals, the channel impulse responses and the propagation losses of the corresponding channels provided by the jammer system, minimize the total jamming power by optimizing the multicarrier radar jamming power allocation while the achieved MI between the radar return and the target impulse response is enforced to be less than a predetermined threshold. These criteria are different from each other in the way the scattering off the target due to the communication signals is considered as useful energy, as interference, or ignored altogether at the radar receiver. The proposed optimization problems in this paper are solved analytically and their solutions represent the optimum power allocation for each subcarrier in the OFDM jamming waveform.

3.2. LPI Based Jamming Power Allocation Criterion 1

For a radar system, the MI between the reflected radar return and the target impulse response should be increased to obtain more information about the target. While as the opponent of a radar, the jammer would like to minimize the MI to protect the target from estimation by utilizing smart noise jamming. In this paper, we adopt the MI as a metric for target parameter estimation performance in the joint radar and communication system. With the derivations in [11], the achievable MI between y and, jointly, \mathbf{h}_r and \mathbf{h}_s , can be written as:

$$\begin{aligned} \text{MI}(\mathbf{y}; \mathbf{h}_r, \mathbf{h}_s) &\triangleq H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r, \mathbf{h}_s) \\ &= H(\mathbf{y}) - H(\mathbf{r}_d + \mathbf{r}_j + \mathbf{n}) \\ &= \sum_{k=0}^{K-1} \log\left(1 + \frac{|X_r[k]|^2 |H_r[k]|^2 L_r[k] + |X_s[k]|^2 |H_s[k]|^2 L_s[k]}{|X_s[k]|^2 L_d[k] + |J[k]|^2 L_j[k] + \sigma_n^2[k]}\right), \end{aligned} \quad (3)$$

where \mathbf{y} represents the vector corresponding to the signal at the radar receiver, \mathbf{r}_d denotes the vector corresponding to the communication signal arriving at the radar on a direct path, \mathbf{r}_j denotes the vector corresponding to the jamming signal arriving at the radar and \mathbf{n} stands for the vector corresponding to the noise. In (3), $|X_r[k]|^2$ and $|X_s[k]|^2$ are the power of the radar and communication signals for the k th subcarrier, respectively. $J[k]$ and $\sigma_n^2[k]$ denote the power of the jammer and the noise for the k th subcarrier, respectively. $H_r[k]$ and $H_s[k]$ are the target impulse responses for the radar-target-radar path and BS-target-radar path, respectively. The propagation losses of the

corresponding channels for the k th subcarrier can be expressed as follows [12][13]:

$$\begin{cases} L_r[k] = \frac{G_t^2 \lambda_k^2}{(4\pi)^3 d_r^4}, \\ L_s[k] = \frac{G_s G_t \lambda_k^2}{(4\pi)^3 d_s^2 d_r^2}, \\ L_d[k] = \frac{G_s G_t \lambda_k^2}{(4\pi)^2 d_b^2}, \\ L_j[k] = \frac{G'_t G_j \lambda_k^2}{(4\pi)^2 d_j^2}, \end{cases} \quad (4)$$

where G_t is the antenna gain of the radar system, G_s is the antenna gain of the communication system, G'_t is the sidelobe antenna gain of the radar system, G_j is the antenna gain of the jammer, and λ_k denotes the wavelength at k th subcarrier. We let d_r , d_s , d_b , and d_j represent the distance between the radar and the target, between the communication system and the target, between the radar and the communication system, and between the jammer and the radar, respectively. It should be pointed out from (3) that the communication signals scattering off the target is considered as useful energy. In this case, we can notice that the achievable MI is related to the jamming waveform, radar transmission waveform, the communication waveform, and the impulse responses of corresponding channels. Intuitively, the minimization of the MI implies that the echo contains as little information of the target as possible, which will lead to poor target parameter estimation performance. However, it also leads to transmitting much more jamming power, which may be in contradiction with the LPI requirement.

Herein, we concentrate on the LPI based adaptive multicarrier jamming power allocation for the joint radar and communication system, whose purpose is to minimize the total jamming power for a predefined target characterization performance. Eventually, the adaptive jamming power allocation strategy based on LPI can be formulated as:

$$\min_{J[k]} \sum_{k=0}^{K-1} |J[k]|^2, \quad (5a)$$

$$s.t. : \begin{cases} \sum_{k=0}^{K-1} \log(1 + \frac{|X_r[k]|^2 |H_r[k]|^2 L_r[k] + |X_s[k]|^2 |H_s[k]|^2 L_s[k]}{|X_s[k]|^2 L_d[k] + |J[k]|^2 L_j[k] + \sigma_n^2[k]}) \leq \gamma_{\max}, \\ 0 \leq |J[k]|^2 \leq P_{\max,k}. \end{cases} \quad (5b)$$

where γ_{\max} denotes the given MI threshold for target estimation performance. The transmitted jamming power for the k th subcarrier is constrained by a maximum value $P_{\max,k}$ and a minimum value 0. After simplifying the constraints and making the notation $x_k = |J[k]|^2$, we can rewrite the optimization problem (5) as:

$$\min_{x_k} \sum_{k=0}^{K-1} x_k, \quad (6a)$$

$$s.t. : \begin{cases} \sum_{k=0}^{K-1} \log(1 + \frac{a_k + b_k}{x_k c_k + d_k}) \leq \gamma_{\max}, \\ 0 \leq \mathbf{x} \leq \mathbf{P}_{\max}. \end{cases} \quad (6b)$$

where we define $a_k = |X_r[k]|^2 |H_r[k]|^2 L_r[k]$, $b_k = |X_s[k]|^2 |H_s[k]|^2 L_s[k]$, $c_k = L_j[k]$, and $d_k = |X_s[k]|^2 L_d[k] + \sigma_n^2[k]$.

Theorem 1: Define

$$\begin{cases} e_k = c_k^2, \\ f_k = c_k(a_k + b_k + 2d_k), \\ g_k = d_k(a_k + b_k + d_k), \\ h_k = c_k(a_k + b_k). \end{cases} \quad (7)$$

The optimal noise jamming power allocation corresponding to problem (6) that minimizes the total transmitting power of the jammer under a predefined MI threshold should satisfy (8):

$$x_k^* = \begin{cases} 0, & \lambda_3^* h_k - g_k \leq 0, \\ -\frac{f_k}{2e_k} + \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k(g_k - \lambda_3^* h_k)}, & 0 < \lambda_3^* h_k - g_k < P_{\max,k}^2 e_k + P_{\max,k} f_k, \\ P_{\max,k}, & \lambda_3^* h_k - g_k \geq P_{\max,k}^2 e_k + P_{\max,k} f_k. \end{cases} \quad (8)$$

λ_3^* is the Lagrange dual variable corresponding to the constraint on the MI constraint:

$$\sum_{k=0}^{K-1} \log\left(1 + \frac{a_k + b_k}{x_k^* c_k + d_k}\right) \leq \gamma_{\max}. \quad (9)$$

Proof: In this paper, we employ the method of Lagrange multipliers to solve constrained optimization problem (6). Introducing Lagrange multipliers $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, and $\lambda_3 \geq 0$ for the multiple constraints, the Lagrange of problem (6) can be equivalently expressed by:

$$\begin{aligned} L(\mathbf{x}, \lambda_1, \lambda_2, \lambda_3) &= \sum_{k=0}^{K-1} x_k + \lambda_1^T (-\mathbf{x}) + \lambda_2^T (\mathbf{x} - \mathbf{P}_{\max}) + \lambda_3 \\ &\times \left[\sum_{k=0}^{K-1} \log\left(1 + \frac{a_k + b_k}{x_k c_k + d_k}\right) - \gamma_{\max} \right]. \end{aligned} \quad (10)$$

In order to obtain the optimal solution, taking the first derivative of (10) with respect to x_k and setting it to zero, we can observe that:

$$\frac{\partial L}{\partial x_k} = 1 - \lambda_{1,k} + \lambda_{2,k} + \frac{\lambda_3}{1 + \frac{a_k + b_k}{x_k c_k + d_k}} \times \frac{-(a_k + b_k)c_k}{(x_k c_k + d_k)^2} = 0. \quad (11)$$

According to the Karush-Kuhn-Tucker (KKT) conditions, if \mathbf{x}^* is the optimal solution, it must satisfy the stationarity condition $\frac{\partial L}{\partial x_k} = 0$, primal feasibility $\sum_{k=0}^{K-1} \log\left(1 + \frac{a_k + b_k}{x_k^* c_k + d_k}\right) \leq \gamma_{\max}$, $\mathbf{0} \leq \mathbf{x}^* \leq \mathbf{P}_{\max}$, dual feasibility $\lambda_1^* \geq \mathbf{0}$, $\lambda_2^* \geq \mathbf{0}$, $\lambda_3^* \geq 0$ and complementary slackness which states that a primal constraint is satisfied with equality, if and only if, the associated dual variable is strictly greater than zero [10]. From the stationary condition, when \mathbf{x}^* is optimal, we obtain:

$$x_k^* = -\frac{f_k}{2e_k} + \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k(g_k - \frac{\lambda_3^* h_k}{1 - \lambda_{1,k}^* + \lambda_{2,k}^*})}. \quad (12)$$

It is apparent from (6) and (12) that the optimality conditions can be separately investigated for three possibilities regarding the optimal allocated power in each subcarrier. At the optimality, each subcarrier can be allocated either with no power ($x_k^* = 0$), with maximum transmitting power ($x_k^* = P_{\max,k}$), or with a power between these two extreme cases ($0 < x_k^* < P_{\max,k}$).

If $0 < x_k^* < P_{\max,k}$, then $\lambda_{1,k}^* = \lambda_{2,k}^* = 0$, we have:

$$\begin{aligned} 0 < -\frac{f_k}{2e_k} + \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k(g_k - \lambda_3^* h_k)} < P_{\max,k} \\ \Leftrightarrow 0 < \lambda_3^* h_k - g_k < P_{\max,k}^2 e_k + P_{\max,k} f_k. \end{aligned} \quad (13)$$

Then, x_k^* can be computed as:

$$x_k^* = -\frac{f_k}{2e_k} + \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k(g_k - \lambda_3^* h_k)}, \quad (14)$$

where λ_3^* is a constant determined by the predetermined MI constraint:

$$\sum_{k=0}^{K-1} \log\left(1 + \frac{a_k + b_k}{x_k^* c_k + d_k}\right) \leq \gamma_{\max}. \quad (15)$$

If $x_k^* = 0$, then $\lambda_{1,k}^* > 0$, $\lambda_{2,k}^* = 0$, we can obtain:

$$\begin{aligned} x_k + \frac{f_k}{2e_k} = \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k\left(g_k - \frac{\lambda_3^* h_k}{1 - \lambda_{1,k}^*}\right)} > \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k(g_k - \lambda_3^* h_k)} \\ \Leftrightarrow \lambda_3^* h_k - g_k < 0. \end{aligned} \quad (16)$$

Then, x_k^* can be given by:

$$x_k^* = 0. \quad (17)$$

If $x_k^* = P_{\max,k}$, then $\lambda_{1,k}^* = 0$, $\lambda_{2,k}^* > 0$, we can have:

$$\begin{aligned} x_k + \frac{f_k}{2e_k} = \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k\left(g_k - \frac{\lambda_3^* h_k}{1 + \lambda_{2,k}^*}\right)} < \frac{1}{2e_k} \sqrt{f_k^2 - 4e_k(g_k - \lambda_3^* h_k)} \\ \Leftrightarrow \lambda_3^* h_k - g_k > P_{\max,k}^2 e_k + P_{\max,k} f_k. \end{aligned} \quad (18)$$

Then, x_k^* is obtained as:

$$x_k^* = P_{\max,k}. \quad (19)$$

Therefore, the optimal jamming power allocation solution can be derived as (8), which completes the proof. ■

In this paper, the well-known bisection search method is utilized to obtain λ_3^* [14][15]. For brevity, the iterative procedure for bisection search is omitted. The iterative procedure of Criterion 1 is detailed in Algorithm 1.

Algorithm 1 : LPI based noise jamming power allocation Criterion 1

- 1: **Initialization:** $\lambda_3 = 0$, γ_{\max} , $P_{\max,k}$, iterative index $t = 1$, the tolerance $\epsilon > 0$;
 - 2: **Loop until:** $\text{MI}^{(t)} - \gamma_{\max} < \epsilon$
 for $k = 1, \dots, K$, do
 Calculate $x_k^{(t)}$ by solving (8);
 Calculate $\text{MI}^{(t)} \leftarrow \sum_{k=0}^{K-1} \log(1 + \frac{a_k + b_k}{x_k^{(t)} c_k + d_k})$;
 Obtain $\lambda_3^{(t+1)}$ via bisection search method;
 Set $t \leftarrow t + 1$;
 end for
 - 3: **End loop**
 - 4: **Update:** Update $x_k^* \leftarrow x_k^{(t)}$ for $\forall k$.
-

3.3. LPI Based Jamming Power Allocation Criterion 2

Next, we define the achievable MI between \mathbf{y} and \mathbf{h}_r as follows:

$$\begin{aligned}
 \text{MI}(\mathbf{y}; \mathbf{h}_r) &\triangleq H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r) \\
 &= H(\mathbf{y}) - H(\mathbf{r}_s + \mathbf{r}_d + \mathbf{r}_j + \mathbf{n}) \\
 &= \sum_{k=0}^{K-1} \log(1 + \frac{|X_r[k]|^2 |H_r[k]|^2 L_r[k]}{|X_s[k]|^2 |H_s[k]|^2 L_s[k] + |X_s[k]|^2 L_d[k] + |J[k]|^2 L_j[k] + \sigma_n^2[k]}), \quad (20)
 \end{aligned}$$

where \mathbf{r}_s denotes the vector corresponding to the communication signals scattered off the target at the radar receiver. One can see from (20) that the scattering off the target due to the communication signal is considered as interference. Similarly, the optimal radar jamming power allocation approach based on LPI can be expressed as:

$$\min_{J[k]} \sum_{k=0}^{K-1} |J[k]|^2, \quad (21a)$$

$$s.t. : \begin{cases} \sum_{k=0}^{K-1} \log(1 + \frac{|X_r[k]|^2 |H_r[k]|^2 L_r[k]}{|X_s[k]|^2 |H_s[k]|^2 L_s[k] + |X_s[k]|^2 L_d[k] + |J[k]|^2 L_j[k] + \sigma_n^2[k]}) \leq \gamma_{\max}, \\ 0 \leq |J[k]|^2 \leq P_{\max,k}. \end{cases} \quad (21b)$$

After simplifying the constraints and using the same notations, we can rewrite the optimization problem (21) as:

$$\min_{x_k} \sum_{k=0}^{K-1} x_k, \quad (22a)$$

$$s.t. : \begin{cases} \sum_{k=0}^{K-1} \log(1 + \frac{a_k}{x_k c_k + b_k + d_k}) \leq \gamma_{\max}, \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{P}_{\max}. \end{cases} \quad (22b)$$

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Content may change prior to final publication in an issue of the journal. To cite the paper please use the doi provided on the Digital Library page.

Theorem 2: Define

$$\begin{cases} m_k = c_k(a_k + 2b_k + 2d_k), \\ n_k = (a_k + b_k + d_k)(b_k + d_k), \\ u_k = a_k c_k. \end{cases} \quad (23)$$

The optimal jamming power allocation corresponding to problem (22) that minimizes the total noise jamming power under a predefined MI threshold should satisfy:

$$x_k^* = \begin{cases} 0, & \lambda_3^* u_k - n_k \leq 0, \\ -\frac{m_k}{2e_k} + \frac{1}{2e_k} \sqrt{m_k^2 - 4e_k(n_k - \lambda_3^* u_k)}, & 0 < \lambda_3^* u_k - n_k < P_{\max,k}^2 e_k + P_{\max,k} m_k, \\ P_{\max,k}, & \lambda_3^* u_k - n_k \geq P_{\max,k}^2 e_k + P_{\max,k} m_k. \end{cases} \quad (24)$$

λ_3^* is the Lagrange dual variable corresponding to the constraint on the MI constraint:

$$\sum_{k=0}^{K-1} \log\left(1 + \frac{a_k}{x_k^* c_k + b_k + d_k}\right) \leq \gamma_{\max}. \quad (25)$$

Proof: We invoke the Lagrange multiplier technique yielding an objective function:

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \lambda_3) &= \sum_{k=0}^{K-1} x_k + \boldsymbol{\lambda}_1^T (-\mathbf{x}) + \boldsymbol{\lambda}_2^T (\mathbf{x} - \mathbf{P}_{\max}) + \lambda_3 \\ &\quad \times \left[\sum_{k=0}^{K-1} \log\left(1 + \frac{x_k a_k}{x_k c_k + b_k + d_k}\right) - \gamma_{\max} \right]. \end{aligned} \quad (26)$$

Taking the first derivative of (26) with respect to x_k , and setting it to zero, we thus obtain:

$$\frac{\partial L}{\partial x_k} = 1 - \lambda_{1,k} + \lambda_{2,k} - \frac{\lambda_3}{1 + \frac{x_k a_k}{x_k c_k + b_k + d_k}} \times \frac{a_k c_k}{(x_k c_k + b_k + d_k)^2} = 0.$$

After basic algebraic manipulations, we can reach the optimal solution x_k^* as a function of the Lagrange multipliers:

$$x_k^* = -\frac{m_k}{2e_k} + \frac{1}{2e_k} \sqrt{m_k^2 - 4e_k(n_k - \frac{\lambda_3^* u_k}{1 - \lambda_{1,k}^* + \lambda_{2,k}^*})}. \quad (27)$$

From complementary slackness, we must consider the following three cases:

If $0 < x_k^* < P_{\max,k}$, then $\lambda_{1,k}^* = \lambda_{2,k}^* = 0$, we have:

$$0 < \lambda_3^* u_k - n_k < P_{\max,k}^2 e_k + P_{\max,k} m_k. \quad (28)$$

Then, x_k^* can be derived as:

$$x_k^* = -\frac{m_k}{2e_k} + \frac{1}{2e_k} \sqrt{m_k^2 - 4e_k(n_k - \lambda_3^* u_k)}, \quad (29)$$

where λ_3^* is a constant determined by the MI constraint:

$$\sum_{k=0}^{K-1} \log\left(1 + \frac{a_k}{x_k^* c_k + b_k + d_k}\right) \leq \gamma_{\max}. \quad (30)$$

If $x_k^* = 0$, then $\lambda_{1,k}^* > 0$, $\lambda_{2,k}^* = 0$, we can obtain:

$$\lambda_3^* u_k - n_k < 0. \quad (31)$$

Then, we have:

$$x_k^* = 0. \quad (32)$$

If $x_k^* = P_{\max,k}$, then $\lambda_{1,k}^* = 0$, $\lambda_{2,k}^* > 0$, we can have:

$$\lambda_3^* u_k - n_k \geq P_{\max,k}^2 e_k + P_{\max,k} m_k. \quad (33)$$

Then, x_k^* is obtained as:

$$x_k^* = P_{\max,k}. \quad (34)$$

Thus, the optimal multicarrier noise jamming power allocation solution can be written as (24), which completes the proof. ■

The iterative procedure of Criterion 2 is similar to Algorithm 1, which is omitted here for brevity.

3.4. LPI Based Jamming Power Allocation Criterion 3

One can also choose the MI expression between \mathbf{y} and \mathbf{h}_r , conditioned on \mathbf{h}_s as shown in (35):

$$\begin{aligned} \text{MI}(\mathbf{y}; \mathbf{h}_r) | \mathbf{h}_s &\triangleq H(\mathbf{y} | \mathbf{h}_s) - H(\mathbf{y} | \mathbf{h}_r, \mathbf{h}_s) \\ &= H(\mathbf{r} + \mathbf{r}_d + \mathbf{r}_j + \mathbf{n}) - H(\mathbf{r}_d + \mathbf{r}_j + \mathbf{n}) \\ &= \sum_{k=0}^{K-1} \log\left(1 + \frac{|X_r[k]|^2 |H_r[k]|^2 L_r[k]}{|X_s[k]|^2 L_d[k] + |J[k]|^2 L_j[k] + \sigma_n^2[k]}\right), \end{aligned} \quad (35)$$

where \mathbf{r} represents the vector corresponding to the radar signals. It can be seen from (35) that the scattering due to the communication signal is ignored. Proceeding as before, we can write the optimization problem as:

$$\min_{J[k]} \sum_{k=0}^{K-1} |J[k]|^2, \quad (36a)$$

$$s.t. : \begin{cases} \sum_{k=0}^{K-1} \log\left(1 + \frac{|X_r[k]|^2 |H_r[k]|^2 L_r[k]}{|X_s[k]|^2 L_d[k] + |J[k]|^2 L_j[k] + \sigma_n^2[k]}\right) \leq \gamma_{\max}, \\ 0 \leq |J[k]|^2 \leq P_{\max,k}. \end{cases} \quad (36b)$$

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After simplifying the constraints and using the same notations, we can rewrite the optimization problem (36) as:

$$\min_{x_k} \sum_{k=0}^{K-1} x_k, \quad (37a)$$

$$s.t. : \begin{cases} \sum_{k=0}^{K-1} \log(1 + \frac{a_k}{x_k c_k + d_k}) \leq \gamma_{\max}, \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{P}_{\max}. \end{cases} \quad (37b)$$

Theorem 3: Define

$$\begin{cases} v_k = c_k(a_k + 2d_k), \\ w_k = d_k(a_k + d_k). \end{cases} \quad (38)$$

The optimal noise jamming power allocation corresponding to problem (37) that minimizes the total noise jamming power under a predefined MI threshold should satisfy (39):

$$x_k^* = \begin{cases} 0, & \lambda_3^* u_k - w_k \leq 0, \\ -\frac{v_k}{2e_k} + \frac{1}{2e_k} \sqrt{v_k^2 - 4e_k(w_k - \lambda_3^* u_k)}, & 0 < \lambda_3^* u_k - w_k < P_{\max,k}^2 e_k + P_{\max,k} v_k, \\ P_{\max,k}, & \lambda_3^* u_k - w_k \geq P_{\max,k}^2 e_k + P_{\max,k} v_k. \end{cases} \quad (39)$$

λ_3^* is the Lagrange dual variable corresponding to the constraint on the MI constraint:

$$\sum_{k=0}^{K-1} \log(1 + \frac{a_k}{x_k^* c_k + d_k}) \leq \gamma_{\max}. \quad (40)$$

Proof: We employ again the approach of Lagrange multipliers as in (10), we obtain the objective function:

$$\begin{aligned} L(\mathbf{x}, \lambda_1, \lambda_2, \lambda_3) &= \sum_{k=0}^{K-1} x_k + \lambda_1^T (-\mathbf{x}) + \lambda_2^T (\mathbf{x} - \mathbf{P}_{\max}) + \lambda_3 \\ &\times \left[\sum_{k=0}^{K-1} \log(1 + \frac{a_k}{x_k c_k + d_k}) - \gamma_{\max} \right]. \end{aligned} \quad (41)$$

Taking the first derivative of (41) with respect to x_k , and setting it to zero, we can obtain:

$$\frac{\partial L}{\partial x_k} = 1 - \lambda_{1,k} + \lambda_{2,k} - \frac{\lambda_3}{1 + \frac{a_k}{x_k c_k + d_k}} \times \frac{a_k c_k}{(x_k c_k + d_k)^2} = 0. \quad (42)$$

Similarly, the optimal solution x_k^* can be given by:

$$x_k^* = -\frac{v_k}{2e_k} + \frac{1}{2e_k} \sqrt{v_k^2 - 4e_k(w_k - \frac{\lambda_3^* u_k}{1 - \lambda_{1,k}^* + \lambda_{2,k}^*})}. \quad (43)$$

We consider the following three cases:

If $0 < x_k^* < P_{\max,k}$, then $\lambda_{1,k}^* = \lambda_{2,k}^* = 0$, we have:

$$0 < \lambda_3^* u_k - w_k < P_{\max,k}^2 e_k + P_{\max,k} v_k. \quad (44)$$

Then, x_k^* can be given as:

$$x_k^* = -\frac{v_k}{2e_k} + \frac{1}{2e_k} \sqrt{v_k^2 - 4e_k(w_k - \lambda_3^* u_k)}, \quad (45)$$

where λ_3^* is determined by:

$$\sum_{k=0}^{K-1} \log\left(1 + \frac{a_k}{x_k^* c_k + d_k}\right) \leq \gamma_{\max}. \quad (46)$$

If $x_k^* = 0$, then $\lambda_{1,k}^* > 0$, $\lambda_{2,k}^* = 0$, we can obtain:

$$\lambda_3^* u_k - w_k < 0. \quad (47)$$

Then, x_k^* can be given as:

$$x_k^* = 0. \quad (48)$$

If $x_k^* = P_{\max,k}$, then $\lambda_{1,k}^* = 0$, $\lambda_{2,k}^* > 0$, we can have:

$$\lambda_3^* u_k - w_k > P_{\max,k}^2 e_k + P_{\max,k} v_k. \quad (49)$$

Then, x_k^* can be given by:

$$x_k^* = P_{\max,k}. \quad (50)$$

Finally, the optimal noise jamming power allocation solution can be obtained as (39), which completes the proof. ■

Similarly, the iterative procedure of Criterion 3 is also omitted here.

Remark 1: The presented jamming power allocation algorithms are formulated and solved analytically, where the method of Lagrange multipliers is adopted to solve the resulting problems. Here, we employ the bisection search method to choose λ_3^* , which offers fast strategy to allow the jammer system to continuously adjust its jamming power to changing conditions. Note that the exhaustive search approach requires $\mathcal{O}(K(\lambda_3^* - \lambda_{3,\min})/\epsilon)$ iterations for the jamming power allocation, while the proposed algorithms have a reduced complexity of $\mathcal{O}(K \log_2[(\lambda_{3,\max} - \lambda_{3,\min})/\epsilon])$, where λ_3^* is constrained by a maximum value $\lambda_{3,\max}$ and a minimum value $\lambda_{3,\min}$. For large numbers of subcarriers and small MI threshold, significant computational savings can be obtained through the use of the presented strategies. For example, a system with $K = 128$, $\epsilon = 0.01$, $\lambda_{3,\min} = 0$, $\lambda_{3,\max} = 10^5$ and $\lambda_3^* = 10^3$ will require only on the order of 2977 iterations with the proposed algorithms while the exhaustive search method requires on the order of 1.28×10^7 iterations. This indicates that the proposed strategies require only 0.024% of the iterations compared to the exhaustive search. The gap goes up rapidly for large numbers of subcarriers and small MI threshold. It is clear that our proposed algorithms exhibit an optimal performance with dramatically reduced complexity.

Remark 2: It is worth mentioning from the SINR term of the objective functions of (3), (20), and (35) that the reflection off the target due to the communication signal contribute to the signal part in (3), to the interference part in (20) and to neither in (35) [11], which leads to different solutions as shown in (8), (24) and (39), respectively. To be specific, in Criterion 1, both the radar transmitted signal and communication signal scattered off the target, that is, $a_k + b_k$ in (7), can be utilized at the radar receiver, which improve the target estimation performance of the radar system. Thus, the most noise jamming power would be transmitted to minimize the achieved MI in this case, and the total transmit noise jamming power of Criterion 1 is much larger than that of the other criteria. However, the scattering off the target due to the communication signal is considered as interference in Criterion 2, as well as the communication signal arriving through a direct line of sight path at the radar receiver, i.e., $b_k + d_k$ in (23). Subsequently, the radar transmit power is increased to be against the interference. Then, more noise jamming should be allocated in subcarriers. It is apparent that Criterion 3 is the special case of Criterion 2, that is, $b_k = 0$ in (38), which transmits the least noise jamming power. As we will see in the following, the solutions utilizing Criterion 2 and Criterion 3 offer superior LPI performance for the jammer compared to the one that exploits Criterion 1 in the joint radar and communication system.

Remark 3: In realistic scenarios, the target spectra corresponding to various incident and scattered directions and polarized types are stored in a database, which can be obtained based on the target-radar/communication BS orientation. Moreover, the OFDM signal exhibits very high peak to average power ratio (PAPR), which is only valid in linear environment conditions. The signal distortion due to nonlinearity will increase the complexity of signal processing and degrade the radar system performance. Therefore, it is necessary to reduce the PAPR and use a linear amplifier with large variation. In this paper, we suppose that the radar transmitted signals and the communication signals due to different paths can be intercepted and perfectly estimated by the jammer. However, the precise knowledge of these parameters is usually unavailable, which is because of the signal distortion due to reflections and multiple paths for different environmental situations or local oscillator's instability. To handle this, the robust strategies can be developed in the presence of parameter uncertainties [6].

4. Simulation results

In this section, numerical simulations are provide to show the enhancement of the LPI performance brought by the jamming power allocation strategies and reveal the relationships between the jamming power allocation results and several factors. We set the joint radar and communication system in a shared spectrum with $K = 128$ channels. The system parameters are given as shown in Table 1. To solve the proposed power allocation problems in (6), (22), and (37), we assume that the jammer system knows the channel impulse responses, the propagation losses of corresponding channels, the communication signals and the radar transmitted signals by sensing itself with a intercept receiver.

Here, it is assumed that the target is illuminated from the front by the radar signals and from the side by the communication signals. As aforementioned, the point target RCS follows a Swerling II model. The target spectra of corresponding channels $H_r[k]$, $H_s[k]$, and $H_e[k]$ are shown in Fig.2, respectively. The communication power is depicted in Fig.3. Figs.4-6 illustrate the LPI based multicarrier radar jamming power allocation results. For all the criteria presented here, we can observe from these figures that the jamming power allocation is determined by the radar transmitted waveform, the communication waveform and the target impulse responses. To minimize the total

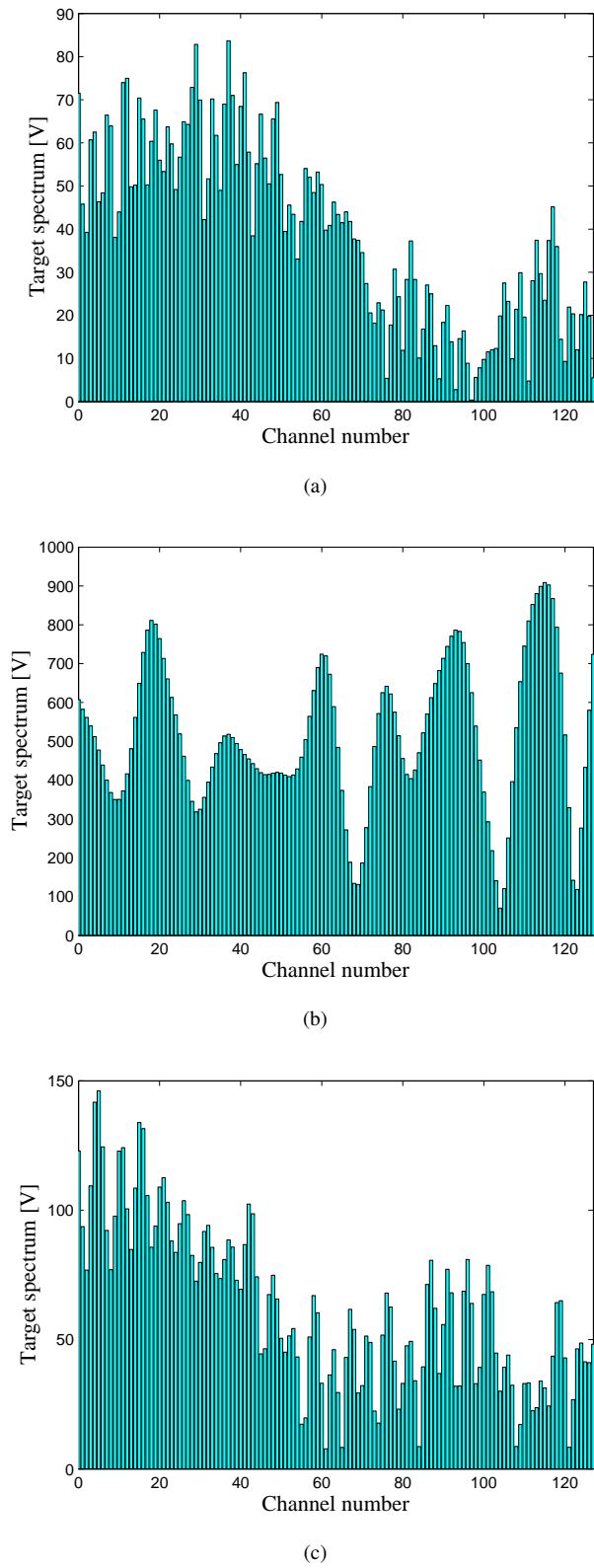


Fig. 2. The target spectra of corresponding channels: (a) $H_r[k]$; (b) $H_s[k]$; (c) $H_e[k]$.

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Table 1 System Parameters

Parameter	Value	Parameter	Value
G_t	30dB	G_s	0dB
G_j	30dB	G'_t	0dB
d_r	20km	d_s	15km
d_b	20km	d_j	50km
$P_{\max,k}$	5000W	γ_{\max}	0.02nats
$\lambda_k(\forall k)$	0.10m	$\sigma_n^2[k](\forall k)$	$1.66 \times 10^{-14}\text{W}$

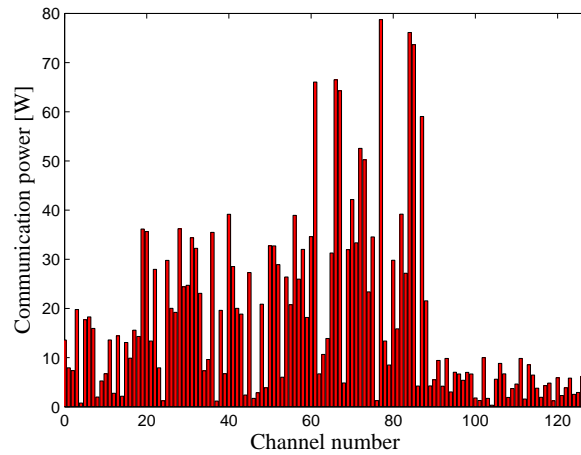
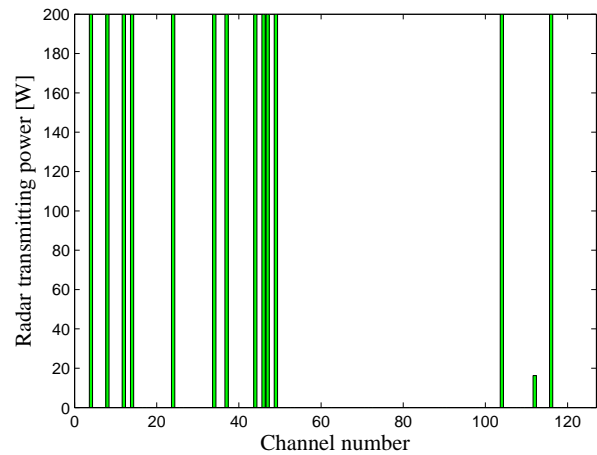
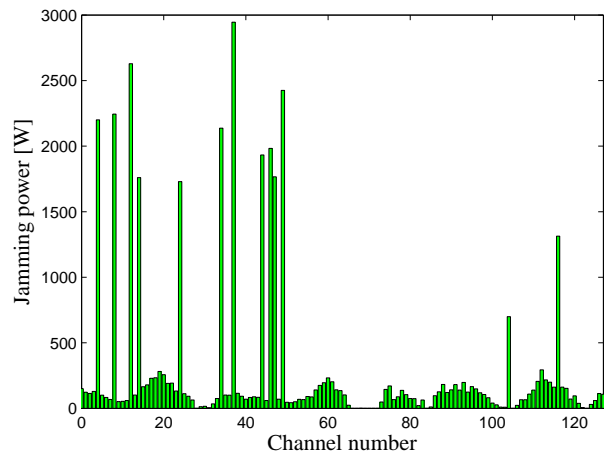


Fig. 3. The communication power.



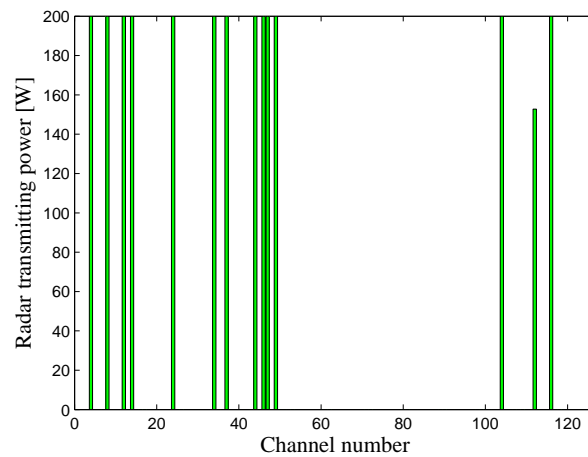
(a)



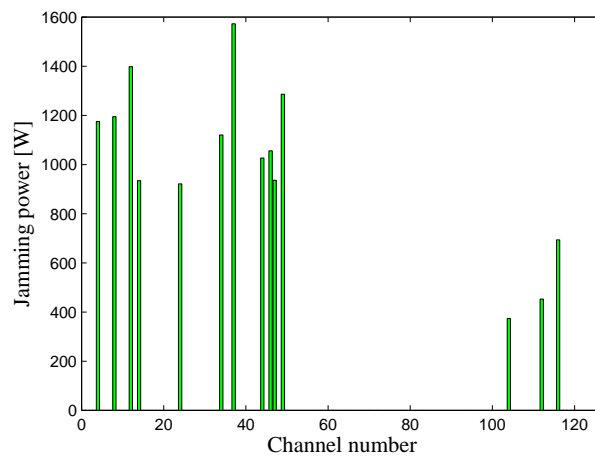
(b)

Fig. 4. LPI based radar jamming power allocation results with Criterion 1: (a) Radar transmission power; (b) Radar jamming power.

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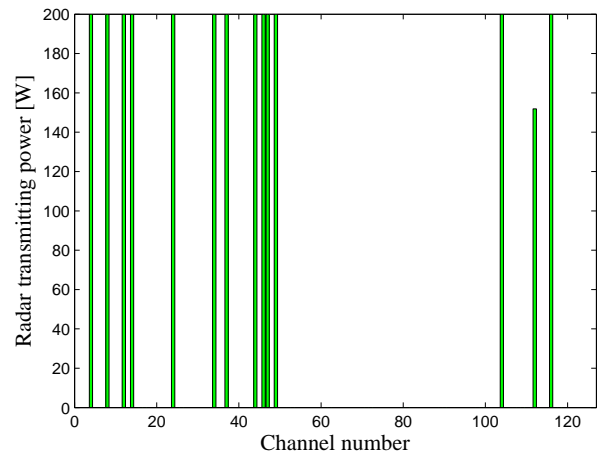


(a)

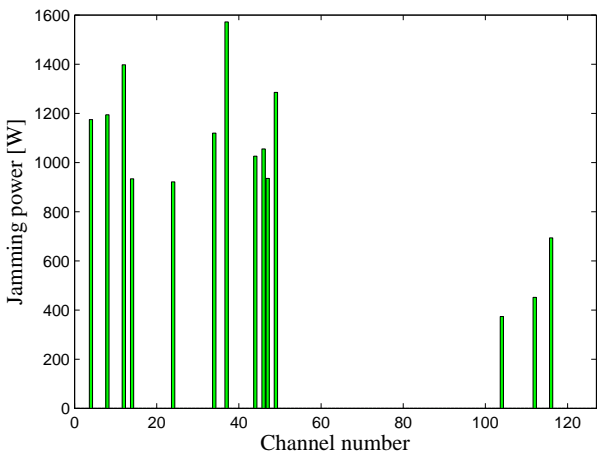


(b)

Fig. 5. LPI based radar jamming power allocation results with Criterion 2: (a) Radar transmission power; (b) Radar jamming power.



(a)



(b)

Fig. 6. LPI based radar jamming power allocation results with Criterion 3: (a) Radar transmission power; (b) Radar jamming power.

jamming power for a predefined MI threshold, the LPI based noise jamming power allocation strategies are formed by the water-filling action, which only place the minimum power over the subcarriers with the largest gain [6][9].

We further see that the jamming power allocation Criterion 1 concentrates its power not only into the channels of the radar waveform, but also into the channels of the communication waveform. This is because that the communication signals scattered off the target would be an important component for target characterization in Criterion 1 [10][11]. Physically speaking, the communication system can provide spatial and signal diversities for the radar in terms of the achievable parameter estimation performance. Intuitively, more jamming power would be distributed into the subcarriers with larger $H_s[k]$. While if the scattering off the target due to communication signals is not considered for the target estimation, the detected energy is reduced, in which case less jamming power is emitted to impair the target parameter estimation performance. Hence, we can conclude that Criterion 2 and Criterion 3 offer superior LPI performance for the jammer compared to the one that exploits Criterion 1 in the joint radar and communication system.

In order to examine the optimality of our strategies on the LPI performance for jammer system, Fig.7 illustrates the comparisons of total jamming power achieved by different algorithms, which are conducted 1000 Monte Carlo trials. The results in Fig.7 indicate that, Criterion 1 transmits the most jamming power when compared with Criterion 2 and Criterion 3, which is because that the radar can make use of the communication signals scattered off the target for a better target estimation capability [11]. Then, the jammer will in turn transmit the largest jamming power to impair the target estimation performance. On the other hand, the jamming power allocation that employs Criterion 3 offers a slightly better LPI performance than the allocation that uses Criterion 2. The reason is that Criterion 3 ignores the scattering off the target due to the communication signals rather than considers it as interference. Equation power allocation method distribute jamming power uniformly over the whole frequency band with no prior knowledge of the radar transmitted signals, the communication signals, and target impulse responses, which has the worst LPI performance. It is obvious that the LPI performance of the jammer system can be significantly improved over that of the equal jamming power allocation method. This further confirms the effectiveness of exploiting our proposed power allocation strategies.

Moreover, clearly from Fig.8, we conclude that the total jamming power under optimal power allocation always comes with the predefined MI threshold. One can observe from Fig.8 that, as the required MI threshold decreases, more jamming power will be transmitted to impair the target parameter estimation performance for radar system, and vice-versa. Generally speaking, the proposed radar jamming power allocation algorithms give insight about the optimum jamming power allocation, which can be realized easily and will undoubtedly improve the LPI performance of the jammer system.

5. Conclusion

This paper proposed three different LPI based adaptive multicarrier radar noise jamming power allocation criteria in a joint radar and communication system assuming that the radar transmitted signals, the communication signals and the power of corresponding channels are known at the jammer. For each criterion, the total noise jamming power is minimized and the resulting problem is solved analytically. Simulation results have been provided to show that the LPI performance of the jammer is significantly improved by utilizing our presented jamming power allocation methods. In future work, we will incorporate the system error models into the presented strategies and look

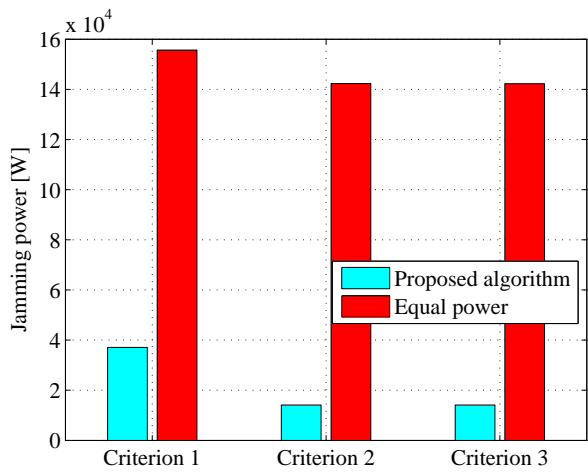


Fig. 7. Comparisons of total jamming power for different algorithms.

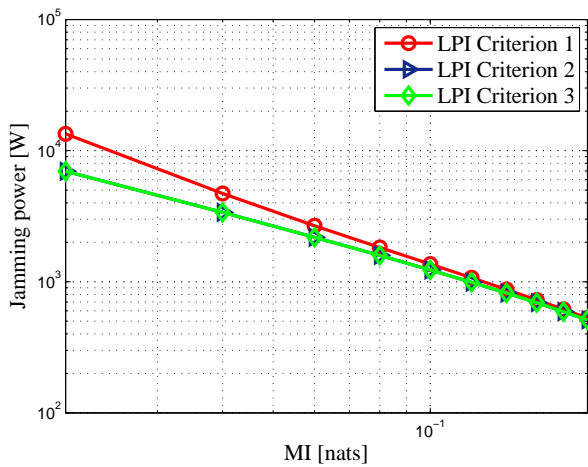


Fig. 8. Jamming power versus achieved MI with various power allocation criteria.

into the problem of robust jamming power allocation for the joint radar and communication system.

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